



F U N D A Ç Ã O  
GETULIO VARGAS

**EPGE**

Escola de Pós-Graduação  
em Economia

## Ensaio Econômico

Escola de

Pós-Graduação

em Economia

da Fundação

Getúlio Vargas

Nº 488

ISSN 0104-8910

### Bounds for the probability distribution function of the linear ACD process

Marcelo Fernandes

Julho de 2003

URL: <http://hdl.handle.net/10438/898>

Os artigos publicados são de inteira responsabilidade de seus autores. As opiniões neles emitidas não exprimem, necessariamente, o ponto de vista da Fundação Getulio Vargas.

ESCOLA DE PÓS-GRADUAÇÃO EM ECONOMIA

Diretor Geral: Renato Fragelli Cardoso

Diretor de Ensino: Luis Henrique Bertolino Braidó

Diretor de Pesquisa: João Victor Issler

Diretor de Publicações Científicas: Ricardo de Oliveira Cavalcanti

Fernandes, Marcelo

Bounds for the probability distribution function of  
the linear ACD process/ Marcelo Fernandes - Rio de Janeiro :  
FGV,EPGE, 2010

(Ensaio Econômico; 488)

Inclui bibliografia.

CDD-330

***Nº 488***

ISSN 0104-8910

***Bounds for the probability distribution function of the linear ACD  
process***

***Marcelo Fernandes***

*Julho de 2003*

# **Bounds for the probability distribution function of the linear ACD process**

**Marcelo Fernandes**

Graduate School of Economics, Fundação Getulio Vargas

Praia de Botafogo, 190, 22253-900 Rio de Janeiro, Brazil

Tel: +55 21 2559 5827

Fax: +55 21 2553 8821

E-mail: [mfernand@fgv.br](mailto:mfernand@fgv.br)

**Abstract:** This paper derives both lower and upper bounds for the probability distribution function of stationary  $ACD(p, q)$  processes. For the purpose of illustration, I specialize the results to the main parent distributions in duration analysis. Simulations show that the lower bound is much tighter than the upper bound.

**JEL Classification:** C22, C41.

**Acknowledgements:** I am grateful to Bernardo de Sá Mota for excellent research assistance and to the CNPQ for supporting this research. The usual disclaimer applies.

# 1 Introduction

The statistical properties of the autoregressive conditional duration model of first order are by now quite well known. Engle and Russell (1998) derive not only the first two moments of the ACD(1,1) model, but also the conditions under which it is stationary and  $\beta$ -mixing. Carrasco and Chen (2002) extend the latter results so as to consider the more general ACD( $p, q$ ) model. Bauwens and Giot (2000) provide a recursive formula to compute the autocorrelation function of the ACD(1,1) process with exponential errors. Fernandes and Grammig (2002) establish conditions for the existence of higher-order moments, strict stationarity and  $\beta$ -mixing property as well as moment recursion relations and autocovariance function of a richer class of nonlinear ACD(1,1) processes that encompasses most autoregressive conditional duration models in the literature.

This note aims at deriving nonasymptotic characterizations of the tail behavior of unconditional distribution function of the ACD( $p, q$ ) process. More precisely, I derive both lower and upper bounds for the probability density function of the duration process. These bounds are quite relevant to the analysis of liquidity risk in that trade and volume durations are intimately related to market activity and liquidity (see Gouriéroux, Jasiak and Le Fol, 1999). Another interesting application relates to actuarial models of credit risk contagion as in Focardi (2001).

The remainder of this paper is organized as follows. Section 2 bounds the probability density function of the ACD( $p, q$ ) process without assuming a particular distribution for the error term. Section 3 sharpens the result by considering the usual specifications of the density of the error term. Section 4 investigate the precision of these bounds through a simple simulation study.

# 2 Bounds

Let  $x_t = \tau_t - \tau_{t-1}$  denote the time spell between two events occurring at times  $\tau_t$  and  $\tau_{t-1}$ . Engle and Russell (1998) propose to account for the serial dependence of financial duration data by formulating an accelerated time process  $x_t = \psi_t \epsilon_t$ ,

where  $\psi_t \equiv E(x_t | \Omega_{t-1})$ ,  $\epsilon_t$  is iid with unity mean, and  $\Omega_{t-1}$  is the set including all information available at time  $\tau_{t-1}$ . The ACD( $p, q$ ) then assumes that  $\psi_t$  and  $\epsilon_t$  are stochastically independent, and

$$\psi_t = \omega + \sum_{i=1}^p \alpha_i x_{t-i} + \sum_{j=1}^q \beta_j \psi_{t-j}, \quad (1)$$

where  $\omega > 0$ ,  $\alpha \equiv (\alpha_1, \dots, \alpha_p) \geq 0$ , and  $\beta \equiv (\beta_1, \dots, \beta_q) \geq 0$ . This parameter restrictions ensure the nonnegativeness of the duration process, whereas imposing  $\gamma \equiv \sum_{i=1}^p \alpha_i + \sum_{j=1}^q \beta_j < 1$  guarantees stationarity.

I take benefit from the close parallel between ACD and GARCH models in order to establish both lower and upper bounds of the probability distribution function. More precisely, I start with a trivial upper bound and then derive a nontrivial lower bound using the techniques put forth by Pawlak and Schmid (2001). Denoting by  $F_\epsilon$  the probability distribution function of the error term, it follows that

$$\begin{aligned} \Pr(x_t \leq z) &= E \left[ \Pr(\psi_t \epsilon_t < z | I_{t-1}) \right] \\ &= E \left[ F_\epsilon \left( \frac{z}{\omega + \sum_{i=1}^p \alpha_i x_{t-i} + \sum_{j=1}^q \beta_j \psi_{t-j}} \right) \right]. \end{aligned} \quad (2)$$

It is readily seen that

$$\Pr(x_t \leq z) \leq F_\epsilon(z/\omega) \quad (3)$$

given that durations are nonnegative as well as  $\alpha$  and  $\beta$ .

As in Pawlak and Schmid (2001), the nontrivial lower bound for the probability distribution function of the duration process holds only for  $z \in [0, \bar{z}]$ , where  $\bar{z}$  varies according to the distribution of the error term. The reason is that, to apply Jensen's inequality to bound (2) from below, one must find the conditions under which the function

$$H(u, v) = F_\epsilon \left( \frac{z}{\omega + \sum_{i=1}^p \alpha_i u_i + \sum_{j=1}^q \beta_j v_j} \right) \quad (4)$$

is convex for all  $u \equiv (u_1, \dots, u_p) \geq 0$  and  $v \equiv (v_1, \dots, v_q) \geq 0$ . It is straightforward to show that the Hessian of (4) is convex if

$$2f_\epsilon(z/M) + \frac{z}{M} f'_\epsilon(z/M) \geq 0 \quad (5)$$

where  $M = \omega + \sum_{i=1}^p \alpha_i u_i + \sum_{j=1}^q \beta_j v_j$ , given that

$$\frac{\partial^2 H(u, v)}{\partial u_r \partial u_s} = \alpha_r \alpha_s \frac{z^2}{M^4} f'_\epsilon(z/M) + 2 \alpha_r \alpha_s \frac{z}{M^3} f_\epsilon(z/M) \quad (6)$$

$$\frac{\partial^2 H(u, v)}{\partial u_r \partial v_s} = \alpha_r \beta_s \frac{z^2}{M^4} f'_\epsilon(z/M) + 2 \alpha_r \beta_s \frac{z}{M^3} f_\epsilon(z/M) \quad (7)$$

$$\frac{\partial^2 H(u, v)}{\partial v_r \partial u_s} = \beta_r \beta_s \frac{z^2}{M^4} f'_\epsilon(z/M) + 2 \beta_r \beta_s \frac{z}{M^3} f_\epsilon(z/M) \quad (8)$$

for  $r \neq s$ . I am now ready to state the main result.

**Theorem.** *Let  $x_t \sim \text{ACD}(p, q)$  satisfying the nonnegativeness and stationarity conditions. Assuming that the density  $f_\epsilon$  of the error term is differentiable then yields that*

$$F_\epsilon(x) \leq \Pr \left( x_t \leq \frac{\omega}{1-\gamma} x \right) \leq F_\epsilon \left( \frac{x}{1-\gamma} \right), \quad (9)$$

where the lower bound holds only for  $x \in [0, (1-\gamma)c]$  with

$$c \equiv \sup_{\tau > 0} \left\{ 2f_\epsilon(x) + x f'_\epsilon(x) \geq 0 \text{ for all } 0 < x < \tau \right\}. \quad (10)$$

**Proof.** It ensues from condition (5) that

$$\begin{aligned} \sup_{\tau > 0} \left\{ 2f_\epsilon(z/M) + \frac{z}{M} f'_\epsilon(z/M) \geq 0 \text{ for all } 0 < x < \tau \right\} \\ = M \sup_{\tau > 0} \{ 2f_\epsilon(x) + x f'_\epsilon(x) \geq 0 \text{ for all } 0 < x < \tau \} = Mc. \end{aligned} \quad (11)$$

The Hessian of (4) is therefore convex if

$$z \leq c \left( \omega + \sum_{i=1}^p \alpha_i u_i + \sum_{j=1}^q \beta_j v_j \right). \quad (12)$$

This is true for any  $\alpha_i, u_i, \beta_j, v_j \geq 0$  if  $z \leq c\omega$ . The result then follows by applying the Jensen's inequality to (2) with  $z = \frac{\omega}{1-\gamma} x$ .  $\blacksquare$

As is apparent, the applicability of the lower bound depends essentially on the constant  $c$ , and hence it is interesting to evaluate (10) for the usual distributions in the ACD literature. I therefore consider in the next section five particular cases, namely exponential, Weibull, Burr, generalized gamma, and uniform. In all instances I normalize the distribution so as to impose unity mean.

### 3 Examples

The ACD modeling aims to match two stylized features in financial duration data, namely serial correlation and overdispersion. Engle and Russell (1998) show indeed that the simple ACD model with exponential errors produce overdispersion. Further, quasi maximum likelihood methods provide consistent estimates for ACD process only if based on the exponential distribution (see Drost and Werker, 2001). It seems therefore natural to start with the exponential assumption for the error term, which yields  $f_E(x) = e^{-x}$  and  $f'_E(x) = -e^{-x}$ . By (5), this implies that  $c = 2$ .

Engle and Russell (1998) argue that the nature of financial durations are more in line with a decreasing baseline hazard rate function. The exponential assumption implies however that the baseline hazard rate function is flat. A natural candidate then is the Weibull distribution with parameter  $\theta$ , which gives rise to either monotonically decreasing ( $0 < \theta < 1$ ) or increasing ( $\theta > 1$ ) baseline hazard rate functions. The density function of the Weibull distribution and its first derivative are

$$f_W(x) = \frac{\theta}{\Gamma(1 + 1/\theta)} x^{\theta-1} \exp \left[ -\frac{x^\theta}{\Gamma(1 + 1/\theta)} \right] \quad (13)$$

$$f'_W(x) = -f_W(x) \left[ \frac{1-\theta}{x} + \frac{\theta}{\Gamma(1 + 1/\theta)} x^{\theta-1} \right], \quad (14)$$

respectively. Condition (5) then becomes

$$f_W(x) \left[ 2 - 1 + \theta - \frac{\theta}{\Gamma(1 + 1/\theta)} x^\theta \right] \geq 0, \quad (15)$$

which implies that  $c = \left[ \Gamma(2 + 1/\theta) \right]^{1/\theta}$ . As a sanity check, observe that in the exponential case  $\theta = 1$ , recovering  $c = \Gamma(3) = 2$ . If one assumes that  $0 < \theta < 1$  as in Engle and Russell (1998), then the sup condition in (10) becomes less stringent than in the exponential context, viz.  $c > 2$ .

Grammig and Maurer (2000) advocate the use of the Burr distribution in order to accommodate more flexible hazard rate functions. Indeed the Burr density

$$f_B(x) = \frac{\theta \xi_B^\theta x^{\theta-1}}{(1 + \kappa \xi_B^\theta x^\theta)^{1+1/\kappa}}, \quad (16)$$



where  $\theta > \kappa > 0$  and

$$\xi_B \equiv \frac{\Gamma(1 + 1/\theta) \Gamma(1/\kappa - 1/\theta)}{\kappa^{1+1/\theta} \Gamma(1 + 1/\kappa)}, \quad (17)$$

entails a nonmonotonic baseline hazard rate function if  $\theta > 1$ . It is easy to show that

$$f'_B(x) = f_B(x) \left[ \frac{\theta - 1}{x} - \left( 1 + \frac{1}{\kappa} \right) \frac{\kappa \theta \xi_B^\theta x^{\theta-1}}{1 + \kappa \xi_B^\theta x^\theta} \right], \quad (18)$$

and hence

$$c = \left( \frac{\theta + 1}{\theta - \kappa} \right)^{1/\theta} \xi_B^{-1} = \left( \frac{\theta + 1}{\theta - \kappa} \right)^{1/\theta} \frac{\kappa^{1+1/\theta} \Gamma(1 + 1/\kappa)}{\Gamma(1 + 1/\theta) \Gamma(1/\kappa - 1/\theta)}. \quad (19)$$

Special cases of the Burr family of distributions are the Weibull ( $\kappa \rightarrow 0$ ), the exponential ( $\kappa \rightarrow 0, \theta = 1$ ), and the log-logistic ( $\theta > \kappa = 1$ ) distributions.

As an alternative to the Burr model, Lunde (1999) puts forward the generalized gamma ACD process, where  $\epsilon_t$  is iid with density

$$f_G(x) = \frac{\xi_G^\theta \theta x^{\theta\kappa-1}}{\Gamma(\kappa)} \exp(-\xi_G^\theta x^\theta) \quad (20)$$

where  $\xi_G \equiv \Gamma(\kappa + 1/\theta)/\Gamma(\kappa)$ . The generalized gamma distribution nests the standard gamma ( $\theta = 1$ ), log-normal ( $\kappa \rightarrow \infty$ ), Weibull ( $\kappa = 1$ ), exponential ( $\theta = \kappa = 1$ ), and half-normal ( $\kappa = 1/2, \theta = 2$ ) distributions.

Although the baseline hazard rate has no closed-form solution, it is possible to derive its shape properties according to the parameter values (Glaser, 1980). If  $\theta\kappa < 1$ , the hazard rate is decreasing for  $\theta \leq 1$ , and U-shaped for  $\theta > 1$ . Conversely, if  $\theta\kappa > 1$ , the hazard rate is increasing for  $\theta \geq 1$ , and inverted U-shaped for  $\theta < 1$ . Lastly, if  $\theta\kappa = 1$ , the hazard rate is decreasing for  $\theta < 1$ , constant for  $\theta = 1$  (exponential case), and increasing for  $\theta > 1$ . The derivative of the density is

$$f'_G(x) = f_G(x) \left[ \frac{\kappa\theta - 1}{x} - \theta \xi_G^\theta x^{\theta-1} \right], \quad (21)$$

yielding

$$c = (\kappa + 1/\theta)^{1/\theta} \frac{\Gamma(\kappa)}{\Gamma(\kappa + 1/\theta)}. \quad (22)$$

It is easy to show that the lower bound is valid for  $c = \sqrt{\pi}$  and  $c = 1 + 1/\kappa$  in the particular cases of the half-normal and standard gamma distributions, respectively.

Finally, albeit the ACD process with  $\epsilon_t \sim U(0, 2)$  does not have much appeal in practice, it entails a very interesting result. Indeed, it turns out that (5) holds for every value of  $x$ , and so the lower bound is always valid. Figure 1 summarizes these results by displaying the constant  $c$  as a function of the distributional parameters. There are no plots for the exponential, half-normal and uniform distributions in view that  $c$  does not vary for them.

## 4 Sharpness of the bounds

To investigate how tight these bounds are, I perform a simple simulation study using an ACD(2,2) process with exponential errors. I set  $\alpha = (0.10, 0.05)$  and  $\beta = (0.45, 0.25)$  and normalize the unconditional expected duration to one by imposing  $\omega = 1 - \gamma$ . Next, I initialize (1) with  $\psi_0 = 1$  and simulate 10,000 realizations of the process and then estimate the unconditional cumulative distribution of the duration process using the empirical distribution of the last 8,000 observations of the sample. Figure 2 illustrates the fact that, despite the slackness of the trivial upper limit, the nontrivial lower bound is extremely sharp and informative. Further simulations show that this result is quite robust to the specification of the linear ACD process. The simulations also indicate that by substituting maximum likelihood estimates for the true values of the parameters, the 95% confidence interval of the lower bound provides a tight confidence band to the true probability distribution function of the process.

## References

- Bauwens, L., Giot, P., 2000, The logarithmic ACD model: An application to the bid-ask quote process of three NYSE stocks, *Annales d'Economie et de Statistique* 60, 117–150.
- Carrasco, M., Chen, X., 2002, Mixing and moment properties of various GARCH and stochastic volatility models, *Econometric Theory* 18, 17–39.
- Drost, F., Werker, B. J. M., 2001, *Semiparametric duration models*, Tilburg University.

- Engle, R. F., Russell, J. R., 1998, Autoregressive conditional duration: A new model for irregularly-spaced transaction data, *Econometrica* 66, 1127–1162.
- Fernandes, M., Grammig, J., 2002, A family of autoregressive conditional duration models, *Ensaio Econômicos* 404, Fundação Getulio Vargas.
- Focardi, S. M., 2001, An actuarial model of credit risk contagion, Discussion Paper 03, Intertek Group.
- Glaser, R. E., 1980, Bathtub and related failure rate characterizations, *Journal of the American Statistical Association* 75, 667–672.
- Gouriéroux, C., Jasiak, J., Le Fol, G., 1999, Intra-day market activity, *Journal of Financial Markets* 2, 193–226.
- Grammig, J., Maurer, K.-O., 2000, Non-monotonic hazard functions and the autoregressive conditional duration model, *Econometrics Journal* 3, 16–38.
- Lunde, A., 1999, A generalized gamma autoregressive conditional duration model, University of Aarhus.
- Pawlak, M., Schmid, W., 2001, On the distributional properties of GARCH processes, *Journal of Time Series Analysis* 22, 339–352.

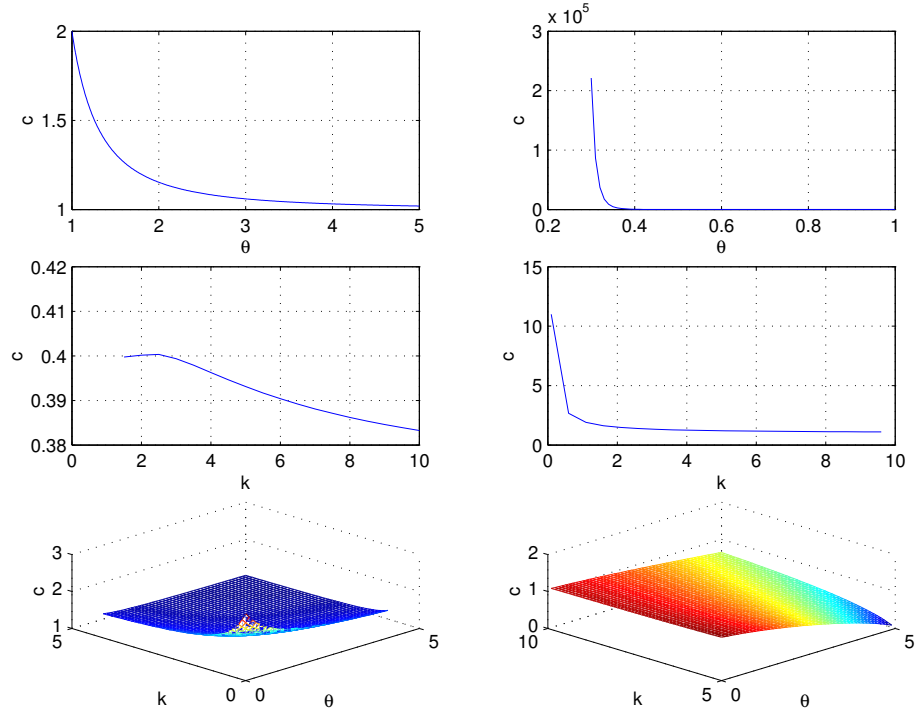


Figure 1: The constant  $c$  as a function of the distributional parameters

coordinates	distribution
(1,1)	Weibull ( $\theta \geq 1$ )
(1,2)	Weibull ( $\theta \leq 1$ )
(2,1)	log-logistic
(2,2)	gamma
(3,1)	generalized gamma
(3,2)	Burr

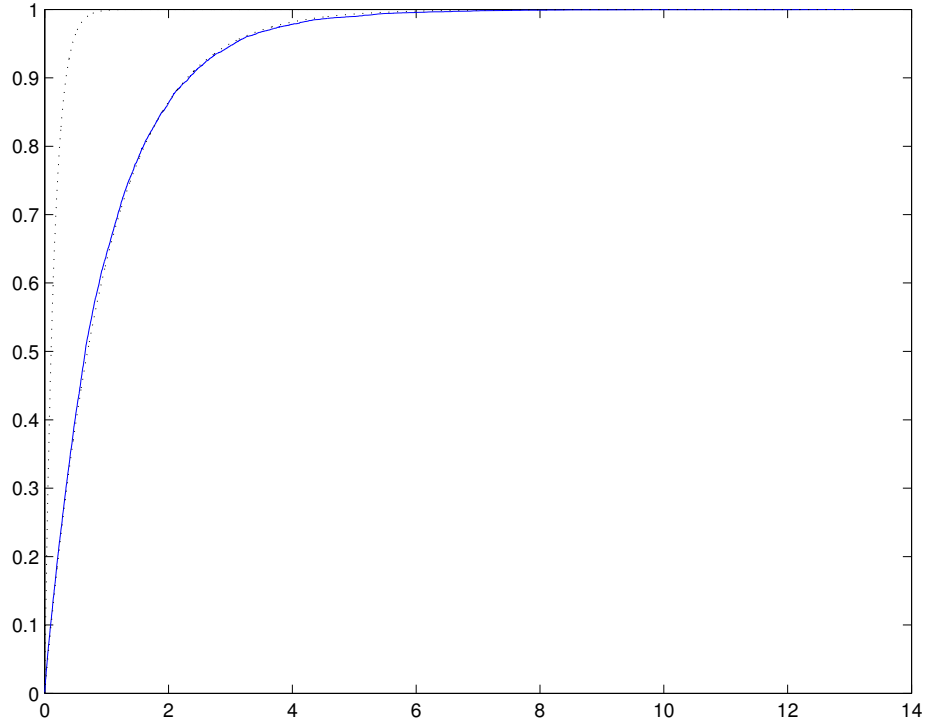


Figure 2: Bounds for the linear ACD probability distribution function

This example refers to an ACD(2,2) process with exponential errors. The parameters are set to  $\alpha = (0.10, 0.05)$ ,  $\beta = (0.45, 0.25)$ ,  $\omega = 1 - \gamma$ , and  $\psi_0 = 1$ . The bounds are based on 10,000 realizations of the process and on the empirical distribution function of the last 8,000 observations of the sample.

# ENSAIOS ECONÔMICOS DA EPGE

443. O MECANISMO MONETÁRIO DE TRANSMISSÃO NA ECONOMIA BRASILEIRA PÓS-PLANO REAL – Marcelo Fernandes; Juan Toro – Abril de 2002 – 33 págs.
444. A ROBUST POVERTY PROFILE FOR BRAZIL USING MULTIPLE DATA SOURCES – Francisco H. G. Ferreira; Peter Lanjouw; Marcelo Neri – Abril de 2002 – 38 págs.
445. THE MISSING LINK: USING THE NBER RECESSION INDICATOR TO CONSTRUCT COINCIDENT AND LEADING INDICES OF ECONOMIC ACTIVITY – João Victor Issler; Farshid Vahid – Maio de 2002 – 31 págs.
446. SPECULATIVE ATTACKS ON DEBTS, DOLLARIZATION AND OPTIMUM CURRENCY AREAS – Aloisio Araujo; Marcia Leon – Maio de 2002 – 74 págs.
447. VINTAGE CAPITAL, DISTORTIONS AND DEVELOPMENT – Samuel de Abreu Pessoa; Rafael Rob – Maio de 2002 – 43 págs.
448. GOVERNMENT ACTIONS TO SUPPORT COFFEE PRODUCERS – AN INVESTIGATION OF POSSIBLE MEASURES FROM THE EUROPEAN UNION – Germán Calfat; Renato G. Flôres Jr. – Junho de 2002 – 30 págs.
449. A MODEL OF CAPITAL ACCUMULATION AND RENT-SEEKING – Paulo Barelli; Samuel de Abreu Pessôa – Junho de 2002 – 50 págs.
450. THE MISSING LINK: USING THE NBER RECESSION INDICATOR TO CONSTRUCT COINCIDENT AND LEADING INDICES OF ECONOMIC ACTIVITY – João Victor Issler; Farshid Vahid – Junho de 2002 – 29 págs.
451. LENDER LIABILITY IN THE CONSUMER CREDIT MARKET – Elisabetta Iossa; Giuliana Palumbo – Agosto de 2002 – 20 págs.
452. DECISION RULES AND INFORMATION PROVISION: MONITORING VERSUS MANIPULATION - Elisabetta Iossa; Giuliana Palumbo – Agosto de 2002 – 37 págs.
453. ON CERTAIN GEOMETRIC ASPECTS OF PORTFOLIO OPTIMISATION WITH HIGHER MOMENTS – Gustavo M. de Athayde; Renato G. Flôres Jr. – Setembro de 2002 – 21 págs.
454. MENSURANDO A PRODUÇÃO CIENTÍFICA INTERNACIONAL EM ECONOMIA DE PESQUISADORES E DEPARTAMENTOS BRASILEIROS - João Victor Issler; Tatiana Caldas de Lima Aché Pillar – Setembro de 2002 – 39 págs.
455. FOREIGN DIRECT INVESTMENT SPILLOVERS: ADDITIONAL LESSONS FROM A COUNTRY STUDY - Renato G. Flores Jr; Maria Paula Fontoura; Rogério Guerra Santos – Setembro de 2002 – 30 págs.
456. A CONTRACTIVE METHOD FOR COMPUTING THE STATIONARY SOLUTION OF THE EULER EQUATION - Wilfredo L. Maldonado; Humberto Moreira – Setembro de 2002 – 14 págs.
457. TRADE LIBERALIZATION AND THE EVOLUTION OF SKILL EARNINGS DIFFERENTIALS IN BRAZIL - Gustavo Gonzaga; Naércio Menezes Filho; Cristina Terra – Setembro de 2002 – 31 págs.

458. DESEMPENHO DE ESTIMADORES DE VOLATILIDADE NA BOLSA DE VALORES DE SÃO PAULO - Bernardo de Sá Mota; Marcelo Fernandes – Outubro de 2002 – 37 págs.
459. FOREIGN FUNDING TO AN EMERGING MARKET: THE MONETARY PREMIUM THEORY AND THE BRAZILIAN CASE, 1991-1998 - Carlos Hamilton V. Araújo; Renato G. Flores Jr. – Outubro de 2002 – 46 págs.
460. REFORMA PREVIDENCIÁRIA: EM BUSCA DE INCENTIVOS PARA ATRAIR O TRABALHADOR AUTÔNOMO - Samantha Taam Dart; Marcelo Côrtes Neri; Flavio Menezes – Novembro de 2002 – 28 págs.
461. DECENT WORK AND THE INFORMAL SECTOR IN BRAZIL – Marcelo Côrtes Neri – Novembro de 2002 – 115 págs.
462. POLÍTICA DE COTAS E INCLUSÃO TRABALHISTA DAS PESSOAS COM DEFICIÊNCIA - Marcelo Côrtes Neri; Alexandre Pinto de Carvalho; Hessia Guilherme Costilla – Novembro de 2002 – 67 págs.
463. SELETIVIDADE E MEDIDAS DE QUALIDADE DA EDUCAÇÃO BRASILEIRA 1995-2001 - Marcelo Côrtes Neri; Alexandre Pinto de Carvalho – Novembro de 2002 – 331 págs.
464. BRAZILIAN MACROECONOMICS WITH A HUMAN FACE: METROPOLITAN CRISIS, POVERTY AND SOCIAL TARGETS – Marcelo Côrtes Neri – Novembro de 2002 – 61 págs.
465. POBREZA, ATIVOS E SAÚDE NO BRASIL - Marcelo Côrtes Neri; Wagner L. Soares – Dezembro de 2002 – 29 págs.
466. INFLAÇÃO E FLEXIBILIDADE SALARIAL - Marcelo Côrtes Neri; Maurício Pinheiro – Dezembro de 2002 – 16 págs.
467. DISTRIBUTIVE EFFECTS OF BRAZILIAN STRUCTURAL REFORMS - Marcelo Côrtes Neri; José Márcio Camargo – Dezembro de 2002 – 38 págs.
468. O TEMPO DAS CRIANÇAS - Marcelo Côrtes Neri; Daniela Costa – Dezembro de 2002 – 14 págs.
469. EMPLOYMENT AND PRODUCTIVITY IN BRAZIL IN THE NINETIES - José Márcio Camargo; Marcelo Côrtes Neri; Maurício Cortez Reis – Dezembro de 2002 – 32 págs.
470. THE ALIASING EFFECT, THE FEJER KERNEL AND TEMPORALLY AGGREGATED LONG MEMORY PROCESSES - Leonardo R. Souza – Janeiro de 2003 – 32 págs.
471. CUSTO DE CICLO ECONÔMICO NO BRASIL EM UM MODELO COM RESTRIÇÃO A CRÉDITO - Bárbara Vasconcelos Boavista da Cunha; Pedro Cavalcanti Ferreira – Janeiro de 2003 – 21 págs.
472. THE COSTS OF EDUCATION, LONGEVITY AND THE POVERTY OF NATIONS - Pedro Cavalcanti Ferreira; Samuel de Abreu Pessoa – Janeiro de 2003 – 31 págs.
473. A GENERALIZATION OF JUDD'S METHOD OF OUT-STEADY-STATE COMPARISONS IN PERFECT FORESIGHT MODELS - Paulo Barelli; Samuel de Abreu Pessoa – Fevereiro de 2003 – 7 págs.
474. AS LEIS DA FALÊNCIA: UMA ABORDAGEM ECONÔMICA - Aloísio Pessoa de Araújo – Fevereiro de 2003 – 25 págs.

475. THE LONG-RUN ECONOMIC IMPACT OF AIDS - Pedro Cavalcanti G. Ferreira; Samuel de Abreu Pessoa – Fevereiro de 2003 – 30 págs.
476. A MONETARY MECHANISM FOR SHARING CAPITAL: DIAMOND AND DYBVIIG MEET KIYOTAKI AND WRIGHT – Ricardo de O. Cavalcanti – Fevereiro de 2003 – 16 págs.
477. INADA CONDITIONS IMPLY THAT PRODUCTION FUNCTION MUST BE ASYMPTOTICALLY COBB-DOUGLAS - Paulo Barelli; Samuel de Abreu Pessoa – Março de 2003 – 4 págs.
478. TEMPORAL AGGREGATION AND BANDWIDTH SELECTION IN ESTIMATING LONG MEMORY - Leonardo R. Souza - Março de 2003 – 19 págs.
479. A NOTE ON COLE AND STOCKMAN - Paulo Barelli; Samuel de Abreu Pessoa – Abril de 2003 – 8 págs.
480. A HIPÓTESE DAS EXPECTATIVAS NA ESTRUTURA A TERMO DE JUROS NO BRASIL: UMA APLICAÇÃO DE MODELOS DE VALOR PRESENTE - Alexandre Maia Correia Lima; João Victor Issler – Maio de 2003 – 30 págs.
481. ON THE WELFARE COSTS OF BUSINESS CYCLES IN THE 20TH CENTURY - João Victor Issler; Afonso Arinos de Mello Franco; Osmani Teixeira de Carvalho Guillén – Maio de 2003 – 29 págs.
482. RETORNOS ANORMAIS E ESTRATÉGIAS CONTRÁRIAS - Marco Antonio Bonomo; Ivana Dall’Agnol – Junho de 2003 – 27 págs.
483. EVOLUÇÃO DA PRODUTIVIDADE TOTAL DOS FATORES NA ECONOMIA BRASILEIRA: UMA ANÁLISE COMPARATIVA - Victor Gomes; Samuel de Abreu Pessoa; Fernando A . Veloso – Junho de 2003 – 45 págs.
484. MIGRAÇÃO, SELEÇÃO E DIFERENÇAS REGIONAIS DE RENDA NO BRASIL - Enestor da Rosa dos Santos Junior; Naércio Menezes Filho; Pedro Cavalcanti Ferreira – Junho de 2003 – 23 págs.
485. THE RISK PREMIUM ON BRAZILIAN GOVERNMENT DEBT, 1996-2002 - André Soares Loureiro; Fernando de Holanda Barbosa - Junho de 2003 – 16 págs.
486. FORECASTING ELECTRICITY DEMAND USING GENERALIZED LONG MEMORY - Lacir Jorge Soares; Leonardo Rocha Souza – Junho de 2003 – 22 págs.
487. USING IRREGULARLY SPACED RETURNS TO ESTIMATE MULTI-FACTOR MODELS: APPLICATION TO BRAZILIAN EQUITY DATA - Álvaro Veiga; Leonardo Rocha Souza – Junho de 2003 – 26 págs.
488. BOUNDS FOR THE PROBABILITY DISTRIBUTION FUNCTION OF THE LINEAR ACD PROCESS – Marcelo Fernandes – Julho de 2003 – 10 págs.
489. CONVEX COMBINATIONS OF LONG MEMORY ESTIMATES FROM DIFFERENT SAMPLING RATES - Leonardo R. Souza; Jeremy Smith; Reinaldo C. Souza – Julho de 2003 – 20 págs.